### Example 2.1: Determine the design flexural strength of the beam cross-section shown below. Use concrete class C20/25 and steel grade S-400

![Cross-sectional diagram]

**Step 1: Design Values (Changing the characteristic value to design value)**

\[
d = 400 - 25 - 8 - 5 = 362\text{mm}
\]

\[
f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c}
\]

\[
f_{yd} = \frac{f_{yk}}{\gamma_S}
\]

**for persistent and transient design situation:**

- \(\gamma_c = 1.5\)
- \(\gamma_S = 1.15\)
- \(\alpha_{cc} = 1\)

**NB:** Take \(\alpha_{cc} = 0.85\)

\[
f_{cd} = \frac{0.85 \times 20}{1.5} = 11.33\text{ MPa}
\]

\[
f_{yd} = \frac{400}{1.15} = 347.83\text{ MPa}
\]

**Step 2: Assume the type of failure**

Assume tension failure with rupture of steel and \(\varepsilon_{cm} \leq \varepsilon_{c2} = 2^\circ/00\)
Cross-Sectional analysis of reinforced concrete beam section for flexure
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Step 3: Draw the strain profile corresponding to the type of failure and use the similarity of triangles to develop a relationship between the unknown strain and the neutral axis.

![Diagram of strain profile]

From Similarity of Triangle

$$\frac{\epsilon_{cm} + 25}{d} = \frac{\epsilon_{cm}}{x} \rightarrow \frac{x}{d} = k_x = \frac{\epsilon_{cm}}{\epsilon_{cm} + 25}$$

(1)

Step 4: Use the equation of alpha corresponding to the assumption in step 2 and the relationship developed in step 3 to calculate the unknown strain.

From equilibrium of forces,

$$C_c = T_s \quad \text{But: } C_c = \alpha_c f_{cd}bd \quad \text{and } T_s = A_s f_{yd}$$

$$C_c = \alpha_c f_{cd}bd = A_s f_{yd}$$

$$\alpha_c = \frac{A_s f_{yd}}{f_{cd}bd}$$

$$\alpha_c = \frac{(2 \times \pi \times 5^2) \times 347.83}{11.33 \times 300 \times 362} = 0.0444$$

(2)

For $$\epsilon_{cm} < \epsilon_{c2}$$, $$\alpha_c = \epsilon_{cm} \left[\frac{6 - \epsilon_{cm}}{12}\right] k_x$$

Substituting $$k_x$$ from Eqn (1),

Examples on analysis of singly reinforced sections
Cross-Sectional analysis of reinforced concrete beam section for flexure

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\[
\alpha_c = \varepsilon_{cm} \left[ \frac{6 - \varepsilon_{cm}}{12} \right] \left( \frac{\varepsilon_{cm}}{\varepsilon_{cm} + 25} \right)
\]

\[
\alpha_c = \left( \frac{6\varepsilon_{cm}^2 - \varepsilon_{cm}^3}{12\varepsilon_{cm} + 300} \right)
\]

\[12\alpha_c\varepsilon_{cm} + 300\alpha_c = 6\varepsilon_{cm}^2 - \varepsilon_{cm}^3\]

\[-\varepsilon_{cm}^3 + 6\varepsilon_{cm}^2 - 12\alpha_c\varepsilon_{cm} - 300\alpha_c = 0\]

Solving the cubic equation results three possible answers

\[\varepsilon_{cm1} = 5.4546 > 3.5 \quad \text{not ok!}\]

\[\varepsilon_{cm2} = -1.3136 < 0 \quad \text{not ok!}\]

\[\varepsilon_{cm3} = 1.859 < 3.5 \quad \text{ok!}\]

Thus, \(\varepsilon_{cm} = 1.859\%\)

**Step 5:** Check if the assumption in step 2 is correct and if it is, proceed to step 8. If the assumption is not correct, repeat step 2 to 5 with another assumption.

\(\varepsilon_{cm} = 1.859 < 2 \quad \text{both of the assumptions are correct!}\)

**Step 6:** Calculate the value of beta

For \(\varepsilon_{cm} < \varepsilon_{c2}\), \(\beta_c = k_x \left[ \frac{8 - \varepsilon_{cm}}{4(6 - \varepsilon_{cm})} \right]\)

\[k_x = \frac{\varepsilon_{cm}}{\varepsilon_{cm} + 25} = \frac{1.859}{1.859 + 25} = 0.0692 \quad \text{and} \quad x = 25.0504 \text{ mm}\]

Substituting the values of \(k_x\) and \(\varepsilon_{cm}\) yields, \(\beta_c = 0.0257\)

**Step 7:** Calculate the moment resistance

\[M = A_s f_{yd} (1 - \beta_c)\]

\[M = (2 \times 5^2) \times 347.83 \times 362 \times (1 - 0.0257) = 19.27 \text{ kNm}\]
Example 2.2: Repeat Example 2.1 replacing 2 Ø 10 by 2 Ø 12

Step 1: Design Values (Changing the characteristic value to design value)

\[ d = 400 - 25 - 8 - 6 = 361 \text{mm} \]

\[ f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c} \]

\[ f_{yd} = \frac{f_{yk}}{\gamma_s} \]

For persistent and transient design situation:

- \( \gamma_c = 1.5 \)
- \( \gamma_s = 1.15 \)

\( \alpha_{cc} = 1 \)

**NB:** Take \( \alpha_{cc} = 0.85 \)

\[ f_{cd} = \frac{0.85 \times 20}{1.5} = 11.33 \text{ MPa} \]

\[ f_{yd} = \frac{400}{1.15} = 347.83 \text{ MPa} \]

Step 2: Assume the type of failure

Assume tension failure with rupture of steel and \( \varepsilon_{cm} \leq \varepsilon_{c2} = 2\% \)}
Cross-Sectional analysis of reinforced concrete beam section for flexure
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Step 3: Draw the strain profile corresponding to the type of failure and use the similarity of triangles to develop a relationship between the unknown strain and the neutral axis.

![Strain profile diagram]

From Similarity of Triangle

\[ \frac{\varepsilon_{cm} + 25}{d} = \frac{\varepsilon_{cm}}{x} \rightarrow \frac{x}{d} = k_x = \frac{\varepsilon_{cm}}{\varepsilon_{cm} + 25} \]  

(1)

Step 4: Use the equation of alpha corresponding to the assumption is step 2 and the relationship developed in step 3 to calculate the unknown strain.

From equilibrium of forces,

\[ C_c = T_s \quad \text{But: } C_c = \alpha_c f_{cd}bd \quad \text{and} \quad T_s = A_s f_{yd} \]

\[ C_c = \alpha_c f_{cd}bd = A_s f_{yd} \]

\[ \alpha_c = \frac{A_s f_{yd}}{f_{cd}bd} \]

\[ \alpha_c = \frac{2 \times \pi \times 6^2 \times 347.83}{11.33 \times 300 \times 361} = 0.064 \]  

(2)

For \( \varepsilon_{cm} < \varepsilon_{c2} \), \( \alpha_c = \varepsilon_{cm} \left( \frac{6 - \varepsilon_{cm}}{12} \right) k_x \)

Substituting \( k_x \) from Eqn (1),

\[ \alpha_c = \varepsilon_{cm} \left( \frac{6 - \varepsilon_{cm}}{12} \right) \left( \frac{\varepsilon_{cm}}{\varepsilon_{cm} + 25} \right) \]

EN 1992
Figure 6.1

NB: The limiting value for \( \varepsilon_s = 25^\circ/oo \) is taken from British National Annex

Examples on analysis of singly reinforced sections
Cross-Sectional analysis of reinforced concrete beam section for flexure

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\[ \alpha_c = \left( \frac{6\varepsilon_{cm}^2 - \varepsilon_{cm}^3}{12\varepsilon_{cm} + 300} \right) \]

\[ 12\alpha_c\varepsilon_{cm} + 300\alpha_c = 6\varepsilon_{cm}^2 - \varepsilon_{cm}^3 \]

\[-\varepsilon_{cm}^3 + 6\varepsilon_{cm}^2 - 12\alpha_c\varepsilon_{cm} - 300\alpha_c = 0 \]

\[-\varepsilon_{cm}^3 + 6\varepsilon_{cm}^2 - 0.768\varepsilon_{cm} - 19.2 = 0 \]

Solving the cubic equation results three possible answers

\[ \varepsilon_{cm1} = 5.116 > 3.5 \quad \text{not ok!} \]

\[ \varepsilon_{cm2} = -1.545 < 0 \quad \text{not ok!} \]

\[ \varepsilon_{cm3} = 2.429 < 3.5 \quad \text{ok!} \]

Thus, \( \varepsilon_{cm} = 2.429 \)%

**Step 5:** Check if the assumption in step 2 is correct and if it is, proceed to step 8. If the assumption is not correct, repeat step 2 to 5 with another assumption.

\[ \varepsilon_{cm} = 2.429 < 3.5 \quad \text{the assumption is correct!} \]

\[ \varepsilon_{cm} = 2.429 > 2 \quad \text{the assumption is not correct!} \]

**Trial 2**

Assume tension failure with rupture of steel and \( \varepsilon_{cm} > \varepsilon_{c2} = 2^\circ / 00 \)

For \( \varepsilon_{cm} > \varepsilon_{c2} \), \( \alpha_c = k_x \left( \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right) \)

Substituting \( k_x \) from Eqn (1),

\[ \alpha_c = \left( \frac{\varepsilon_{cm}}{\varepsilon_{cm} + 25} \right) \left( \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right) \]

\[ 3\alpha_c\varepsilon_{cm} + 75\alpha_c = 3\varepsilon_{cm} - 2 \]

\[ \varepsilon_{cm} = \frac{-2 - 75\alpha_c}{3\alpha - 3} = \frac{-2 - 75 \times 0.064}{3 \times 0.064 - 3} = 2.42 \]

\[ \varepsilon_{cm} = 2.429 > 2 \quad \text{the assumption is correct!} \]

\[ \varepsilon_{cm} = 2.429 < 3.5 \quad \text{the assumption is correct!} \]
Cross-Sectional analysis of reinforced concrete beam section for flexure
Prepared by: Concrete materials and structures chair

Step 6: Calculate the value of beta

For $\varepsilon_{cm} > \varepsilon_{c2}$, $\beta_c = k_x \left[ \frac{(3\varepsilon_{cm} - 4) + 2}{2\varepsilon_{cm} (3\varepsilon_{cm} - 2)} \right]$

$$k_x = \frac{\varepsilon_{cm}}{\varepsilon_{cm} + 25} = \frac{2.42}{2.42 + 25} = 0.088 \text{ and } x = 31.826\ mm$$

Substituting the values of $k_x$ and $\varepsilon_{cm}$ yields, $\beta_c = 0.0343$

Step 7: Calculate the moment resistance

$$M = A_s f_y d (1 - \beta_c)$$

$$M = (2 \times 6^2) \times 347.83 \times 361 \times (1 - 0.0343) = 27.43\ kNm$$
### Cross-Sectional analysis of reinforced concrete beam section for flexure
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<table>
<thead>
<tr>
<th>Example 2.3: Repeat Example 2.1 replacing $2 , \varnothing 10$ by $4 , \varnothing 14$</th>
</tr>
</thead>
</table>

**Step 1:** Design Values (Changing the characteristic value to design value)

\[
d = 400 - 25 - 8 - 7 = 360 \, mm
\]

\[
f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_C}
\]

\[
f_{yd} = \frac{f_{yk}}{\gamma_S}
\]

for persistent and transient design situation:

- $\gamma_C = 1.5$
- $\gamma_S = 1.15$

$\alpha_{cc} = 1$

**NB:** Take $\alpha_{cc} = 0.85$

\[
f_{cd} = \frac{0.85 \times 20}{1.5} = 11.33 \, MPa
\]

\[
f_{yd} = \frac{400}{1.15} = 347.83 \, MPa
\]

**Step 2:** Assume the type of failure

Assume tension failure with rupture of steel and $\varepsilon_{cm} > \varepsilon_{c2} = \frac{2}{0} \, /\, 0$

---

**Examples on analysis of singly reinforced sections**
Cross-Sectional analysis of reinforced concrete beam section for flexure
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### Step 3: Draw the strain profile corresponding to the type of failure and use the similarity of triangles to develop a relationship between the unknown strain and the neutral axis.

![Strain profile diagram]

From Similarity of Triangle

\[
\frac{\varepsilon_{cm} + 25}{d} = \frac{\varepsilon_{cm}}{x} \quad \Rightarrow \quad \frac{x}{d} = k_x = \frac{\varepsilon_{cm}}{\varepsilon_{cm} + 25}
\]  

### Step 4: Use the equation of alpha corresponding to the assumption is step 2 and the relationship developed in step 3 to calculate the unknown strain.

From equilibrium of forces,

\[ C_c = T_s \quad \text{But: } C_c = \alpha_c f_{cd} b d \quad \text{and} \quad T_s = A_s f_{yd} \]

\[ C_c = \alpha_c f_{cd} b d = A_s f_{yd} \]

\[ \alpha_c = \frac{A_s f_{yd}}{f_{cd} b d} \]

\[ \alpha_c = \left( \frac{4 \times \pi \times 7^2}{11.33 \times 300 \times 360} \right) = 0.175 \]  

For \( \varepsilon_{cm} > \varepsilon_{o2} \), \( \alpha_c = k_x \left( \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right) \)

Substituting \( k_x \) from Eqn (1),

\[ \alpha_c = \left( \frac{\varepsilon_{cm}}{\varepsilon_{cm} + 25} \right) \left( \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right) \]

---

**EN 1992**

**Figure 6.1**

**NB:** The limiting value for \( \varepsilon_s = 25^\circ/\text{o} \) is taken from British National Annex
Cross-Sectional analysis of reinforced concrete beam section for flexure
Prepared by: Concrete materials and structures chair

\[ \alpha_c = \left( \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm} + 75} \right) \]

\[ 3\alpha_c \varepsilon_{cm} + 75\alpha_c = 3\varepsilon_{cm} - 2 \]

\[ \varepsilon_{cm} = \frac{-2 - 75\alpha_c}{3\alpha - 3} = \frac{-2 - 75 \times 0.175}{3 \times 0.175 - 3} = 6.11 \]

**Step 5:** Check if the assumption in step 2 is correct and if it is, proceed to step 8. If the assumption is not correct, repeat step 2 to 5 with another assumption.

\[ \varepsilon_{cm} = 6.11 > 3.5 \quad \text{the assumption is not correct!} \]

**Trial 2**

Assume tension failure with crushing of concrete

![Diagram of cross-sectional analysis](image)

From Similarity of Triangle

\[ \frac{3.5 + \varepsilon_s}{d} = \frac{3.5}{x} \quad \Rightarrow \quad \frac{x}{d} = k_x = \frac{3.5}{3.5 + \varepsilon_s} \]

(1)

For \( \varepsilon_{cm} > \varepsilon_{c2} \), \( \alpha_c = k_x \left( \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right) \)

Substituting \( k_x \) from Eqn (1),

\[ \alpha_c = \left( \frac{3.5}{3.5 + \varepsilon_s} \right) \left( \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right) \]

\[ \alpha_c = \left( \frac{3.5}{3.5 + \varepsilon_s} \right) \left( \frac{3 \times 3.5 - 2}{3 \times 3.5} \right) \]
Cross-Sectional analysis of reinforced concrete beam section for flexure

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\[
\alpha_c = \left(\frac{8.5}{10.5 + 3\varepsilon_s}\right)
\]

\[
10.5\alpha_c + 3\varepsilon_s\alpha_c = 8.5
\]

\[
\varepsilon_s = \frac{8.5 - 10.5\alpha_c}{3\alpha_c} = \frac{8.5 - 10.5 \times 0.175}{3 \times 0.175} = 12.69
\]

\[
\varepsilon_{yd} = \frac{f_{yd}}{E_s} = \frac{347.83}{200 \times 10^3} = 1.74
\]

\[
\varepsilon_s = 12.69 > 1.74 \quad \text{the assumption is correct!}
\]

\[
\varepsilon_s = 12.69 < 25 \quad \text{the assumption is correct!}
\]

**Step 6:** Calculate the value of beta

For \(\varepsilon_{cm} > \varepsilon_{c2}\), \(\beta_c = k_x \left[\frac{\varepsilon_{cm}(3\varepsilon_{cm} - 4) + 2}{2\varepsilon_{cm}(3\varepsilon_{cm} - 2)}\right]\)

\[
k_x = \frac{3.5}{3.5 + \varepsilon_s} = \frac{3.5}{3.5 + 12.69} = 0.216 \quad \text{and} \quad x = 77.76 mm
\]

Substituting the values of \(k_x\) and \(\varepsilon_{cm}\) yields, \(\beta_c = 0.09014\)

**Step 7:** Calculate the moment resistance

\[
M = A_s f_{yd} d (1 - \beta_c)
\]

\[
M = (4 \times 7^2) \times 347.83 \times 360 \times (1 - 0.09014) = 70.15 kNm
\]
**Example 2.4: Repeat Example 2.1 replacing 2 Ø 10 by 5 Ø 24**

![Concrete beam section diagram]

**Step 1: Design Values (Changing the characteristic value to design value)**

\[ d = 400 - 25 - 8 - 12 = 355 \text{ mm} \]

\[ f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c} \]

\[ f_{yd} = \frac{f_{yk}}{\gamma_s} \]

**for persistent and transient design situation:**

✓ \( \gamma_c = 1.5 \)

✓ \( \gamma_s = 1.15 \)

\( \alpha_{cc} = 1 \)

**NB:** Take \( \alpha_{cc} = 0.85 \)

\[ f_{cd} = \frac{0.85 \times 20}{1.5} = 11.33 \text{ MPa} \]

\[ f_{yd} = \frac{400}{1.15} = 347.83 \text{ MPa} \]

**Step 2: Assume the type of failure**

Assume tension failure with crushing of concrete

---

**Cross-Sectional analysis of reinforced concrete beam section for flexure**

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**Reinforced Concrete I**

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Examples on analysis of singly reinforced sections
### Step 3: Draw the strain profile corresponding to the type of failure and use the similarity of triangles to develop a relationship between the unknown strain and the neutral axis.

![Strain Profile Diagram](image)

\[ \frac{3.5 + \varepsilon_s}{d} = \frac{3.5}{x} \Rightarrow \frac{x}{d} = k_x = \frac{3.5}{3.5 + \varepsilon_s} \]  

From Similarity of Triangle

\[ \frac{3.5 + \varepsilon_s}{d} = \frac{3.5}{x} \Rightarrow \frac{x}{d} = k_x = \frac{3.5}{3.5 + \varepsilon_s} \]  

\[ (1) \]

### Step 4: Use the equation of alpha corresponding to the assumption in step 2 and the relationship developed in step 3 to calculate the unknown strain

From equilibrium of forces,

\[ C_c = T_s \quad \text{But:} \quad C_c = \alpha_c f_{cd} bd \quad \text{and} \quad T_s = A_s f_{yd} \]

\[ C_c = \alpha_c f_{cd} bd = A_s f_{yd} \]

\[ \alpha_c = \frac{A_s f_{yd}}{f_{cd} bd} \]

\[ \alpha_c = \frac{(5 \times \pi \times 12^2) \times 347.83}{11.33 \times 300 \times 355} = 0.652 \]  

\[ (2) \]

For \( \varepsilon_{cm} > \varepsilon_{y2} \), \( \alpha_c = k_x \left( \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right) \)

Substituting \( k_x \) from Eqn (1),

\[ \alpha_c = \left( \frac{3.5}{3.5 + \varepsilon_s} \right) \left( \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right) \]
Cross-Sectional analysis of reinforced concrete beam section for flexure

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\[
\alpha_c = \left( \frac{3.5}{3.5 + \varepsilon_s} \right) \left( \frac{3 \times 3.5 - 2}{3 \times 3.5} \right)
\]

\[
\alpha_c = \left( \frac{8.5}{10.5 + 3\varepsilon_s} \right)
\]

10.5\alpha_c + 3\varepsilon_s\alpha_c = 8.5

\[
\varepsilon_s = \frac{8.5 - 10.5\alpha_c}{3\alpha_c} = \frac{8.5 - 10.5 \times 0.652}{3 \times 0.652} = 0.846
\]

**Step 5:** Check if the assumption in step 2 is correct and if it is, proceed to step 8. If the assumption is not correct, repeat step 2 to 5 with another assumption.

\[
\varepsilon_{yd} = \frac{f_{yd}}{E_s} = \frac{347.83}{200 \times 10^3} = 1.74
\]

\[
\varepsilon_s = 0.846 < \varepsilon_{yd} \quad \text{the assumption is not correct!}
\]

Trial 2

Assume compression failure with crushing of concrete

\[
\varepsilon_s < \varepsilon_{yd}
\]

From Similarity of Triangle

\[
\frac{3.5 + \varepsilon_s}{d} = \frac{3.5}{x} \Rightarrow \frac{x}{d} = k_x = \frac{3.5}{3.5 + \varepsilon_s}
\]

For \(\varepsilon_{cm} > \varepsilon_{c2}\), \(\alpha_c = k_x \left( \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right)\)
Cross-Sectional analysis of reinforced concrete beam section for flexure

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\[ \alpha_c = \frac{A_s E_s \varepsilon_s}{f_{cd} b d} \]

\[ \alpha_c = \left( \frac{5 \times \pi \times 12^2}{11.33 \times 300 \times 355} \right) = 0.375 \varepsilon_s \]

(2)

For \( \varepsilon_{cm} > \varepsilon_{c2} \), \( \alpha_c = k_x \left( \frac{3 \varepsilon_{cm} - 2}{3 \varepsilon_{cm}} \right) \)

Substituting \( k_x \) from Eqn (1),

\[ \alpha_c = \left( \frac{3.5}{3.5 + \varepsilon_s} \right) \left( \frac{3 \varepsilon_{cm} - 2}{3 \varepsilon_{cm}} \right) \]

\[ \alpha_c = \left( \frac{3.5}{3.5 + \varepsilon_s} \right) \left( \frac{3 \times 3.5 - 2}{3 \times 3.5} \right) \]

\[ \alpha_c = \left( \frac{8.5}{10.5 + 3 \varepsilon_s} \right) = 0.375 \varepsilon_s \]

\( 10.5 \times 0.375 \varepsilon_s + 3 \times 0.375 \varepsilon_s^2 = 8.5 \)

\( 1.125 \varepsilon_s^2 + 3.9375 \varepsilon_s - 8.5 = 0 \)

Solving the quadratic equation results two possible answers

\( \varepsilon_{s1} = 1.508 > 0 \) \( \ldots \ldots \ldots \ldots \ldots \) ok!

\( \varepsilon_{s2} = -5.008 < 0 \) \( \ldots \ldots \ldots \ldots \ldots \) not ok!

Thus,

\( \varepsilon_s = 1.508 < 1.74 \) \( \text{the assumption is correct!} \)
### Step 6: Calculate the value of beta

For $\varepsilon_{cm} > \varepsilon_{c2}$,

$$\beta_c = k_x \left[ \frac{\varepsilon_{cm} (3\varepsilon_{cm} - 4) + 2}{2\varepsilon_{cm} (3\varepsilon_{cm} - 2)} \right]$$

$$k_x = \frac{3.5}{3.5 + \varepsilon_s} = \frac{3.5}{3.5 + 1.508} = 0.699 \text{ and } x = 248.1 \text{ mm}$$

Substituting the values of $k_x$ and $\varepsilon_{cm}$ yields $\beta_c = 0.2912$

### Step 7: Calculate the moment resistance

$$M = A_s f_{yd} (1 - \beta_c)$$

$$M = (5 \times 12^2) \times 347.83 \times 355 \times (1 - 0.2912) = 171.31 kNm$$
Example 2.5: Find the reinforcement amount which results balanced failure & calculate the moment capacity for the cross section described in Example 2.1. (Assume the effective depth to be 354 mm)

Step 1: Design Values (Changing the characteristic value to design value)

\[ f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c} \]
\[ f_{yd} = \frac{f_{yk}}{\gamma_S} \]

for persistent and transient design situation:
- \(\gamma_c = 1.5\)
- \(\gamma_S = 1.15\)
\(\alpha_{cc} = 1\)

**NB:** Take \(\alpha_{cc} = 0.85\)

\[ f_{cd} = \frac{0.85 \times 20}{1.5} = 11.33 \text{ MPa} \]
\[ f_{yd} = \frac{400}{1.15} = 347.83 \text{ MPa} \]

Step 2: Strain profile for balanced failure

\[ \varepsilon_s = \varepsilon_{yd} \]

From similarity of triangles
\[ \frac{3.5}{x_b} = \frac{\varepsilon_{yd}}{d - x_b} \quad \text{but} \quad \varepsilon_{yd} = 1.74 \]
Cross-Sectional analysis of reinforced concrete beam section for flexure

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<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3.5}{x_b} = \frac{1.74}{d - x_b} )</td>
<td>( x_b ) and ( d ) are coordinates of the section.</td>
</tr>
<tr>
<td>( k_x = \frac{3.5}{3.5 + 1.74} = 0.668 )</td>
<td>Moment of inertia about the neutral axis.</td>
</tr>
<tr>
<td>( x = 236.45 \text{ mm} )</td>
<td>Neutral axis location.</td>
</tr>
<tr>
<td>( \alpha_c = 0.668 \left( \frac{3 \times 3.5 - 2}{3 \times 3.5} \right) = 0.5408 )</td>
<td>Ultimate strain ratio.</td>
</tr>
</tbody>
</table>

**Step 3:** Evaluating \( A_s \)

\[
T_s = A_s f_{yd}
\]

\[
C_c = \alpha_c f_{cd} b d
\]

\[
C_c = T_s
\]

\[
\alpha_c f_{cd} b d = A_s f_{yd}
\]

\[
A_s = \frac{\alpha_c f_{cd} b d}{f_{yd}}
\]

\[
A_s = \frac{0.508 \times 11.33 \times 300 \times 354}{347.83}
\]

\[
A_s = 1870.79 \text{ mm}^2
\]

**Step 4:** Calculate the value of beta

\[
\beta_c = k_x \left[ \frac{\varepsilon_{cm} (3 \varepsilon_{cm} - 4) + 2}{2 \varepsilon_{cm} (3 \varepsilon_{cm} - 2)} \right]
\]

Substituting the values of \( k_x \) and \( \varepsilon_{cm} \) yields \( \beta_c = 0.278 \)

**Step 5:** Calculate the moment resistance

\[
M = A_s f_{yd} d \left( 1 - \beta_c \right)
\]

\[
M = 1870.79 \times 347.83 \times 354 \times (1 - 0.278) = 166.35 \text{ kNm}
\]
Example 2.6: for the cross section described in Example 2.1, Find the reinforcement amount & calculate the moment capacity if $k_x = 0.448$. (Assume the effective depth to be 354 mm)

**Step 1: Design Values (Changing the characteristic value to design value)**

\[ f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c} \]

\[ f_{yd} = \frac{f_{y_k}}{\gamma_s} \]

For persistent and transient design situation:

- $\gamma_c = 1.5$
- $\gamma_s = 1.15$
- $\alpha_{cc} = 1$

**NB:** Take $\alpha_{cc} = 0.85$

\[ f_{cd} = \frac{0.85 \times 20}{1.5} = 11.33 \text{ MPa} \]

\[ f_{yd} = \frac{400}{1.15} = 347.83 \text{ MPa} \]

**Step 2: Strain profile**

From similarity of triangles

\[ \frac{3.5}{x} = \frac{\varepsilon_s}{d - x} \quad \text{but} \quad \varepsilon_{yd} = 1.74 \]
Cross-Sectional analysis of reinforced concrete beam section for flexure
Prepared by: Concrete materials and structures chair

\[ \varepsilon_s = \frac{3.5(d - x)}{x} = 3.5 \left( \frac{1}{k_x} - 1 \right) = 3.5 \left( \frac{1}{0.448} - 1 \right) = 4.3125 \]

For \( \varepsilon_{cm} > \varepsilon_{c2} \), \( \alpha_c = k_x \left( \frac{3 \varepsilon_{cm} - 2}{3 \varepsilon_{cm}} \right) \)

\[ \alpha_c = 0.448 \left( \frac{3 \cdot 3.5 - 2}{3 \cdot 3.5} \right) = 0.363 \]

**Step 3:** Evaluating \( A_s \)

\[ T_s = A_s f_{yd} \]
\[ C_c = \alpha_c f_{cd} b_d \]
\[ C_c = T_s \]
\[ \alpha_c f_{cd} b_d = A_s f_{yd} \]
\[ A_s = \frac{\alpha_c f_{cd} b_d}{f_{yd}} \]

\[ A_s = \frac{0.363 \cdot 11.33 \cdot 300 \cdot 354}{347.83} \]
\[ A_s = 1255.72 \text{ mm}^2 \]

**Step 4:** Calculate the value of beta

For \( \varepsilon_{cm} > \varepsilon_{c2} \), \( \beta_c = k_x \left( \frac{\varepsilon_{cm} (3 \varepsilon_{cm} - 4) + 2}{2 \varepsilon_{cm} (3 \varepsilon_{cm} - 2)} \right) \)

Substituting the values of \( k_x \) and \( \varepsilon_{cm} \) yields, \( \beta_c = 0.1864 \)

**Step 5:** Calculate the moment resistance

\[ M = A_s f_{yd} d (1 - \beta_c) \]

\[ M = 1255.72 \times 347.83 \times 354 \times (1 - 0.1864) = 125.8 \text{ kNm} \]
### Summary

<table>
<thead>
<tr>
<th>Example</th>
<th>$A_s$ (mm²)</th>
<th>$x$ (mm)</th>
<th>$\varepsilon_{cm}$ %</th>
<th>$\varepsilon_{s}$ %</th>
<th>$M_{max}$</th>
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<td>4.31</td>
<td>125.8</td>
</tr>
</tbody>
</table>

![Graph showing moment resistance vs. area with ductile and brittle regions marked](image)

Examples on analysis of singly reinforced sections