Example 2.9: An interior T beam in a floor system has a center to center span of 5.5 m and cross section shown in the figure. The concrete and steel grades are C20/25 and S400 respectively. Compute the design resistance capacity of this beam in the positive moment region. (Cover to stirrup = 25 mm and stirrup of diameter 8 c/c 200 was provided)

Step 1: Design Values (Changing the characteristic value to design value)

\[ d = 500 - (25 - 8 - 10) = 457 \text{ mm} \]

\[ f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c} \]

\[ f_{yd} = \frac{f_{yk}}{\gamma_s} \]

for persistent and transient design situation:

- \( \gamma_c = 1.5 \)
- \( \gamma_S = 1.15 \)

\[ \alpha_{cc} = 1 \]

NB: Take \( \alpha_{cc} = 0.85 \)

\[ f_{cd} = \frac{0.85 \times 20}{1.5} = 11.33 \text{ MPa} \]

\[ f_{yd} = \frac{400}{1.15} = 347.83 \text{ MPa} \]
Step 2: Determine the effective flange width of the beam

The effective flange width $b_{\text{eff}}$ for a T beam or L beam may be derived as:

$$b_{\text{eff}} = \sum b_{\text{eff},i} + b_w \leq b$$

where

$$b_{\text{eff},i} = 0.2b_i + 0.1l_0 \leq 0.2l_0$$

and

$$b_{\text{eff},i} \leq b_i$$

For the given beam, $b_1 = \frac{2.7}{2} = 1.35\text{m}$ and $b_2 = \frac{3.25}{2} = 1.625\text{m}$

Since the beam is an interior beam and the design moment resistance is required to be estimated for the positive moment region,

$$l_0 = 0.7l_2 = 0.7 \times 5.5 = 3.85\text{m}$$

Thus,

$$b_{\text{eff},1} = 0.2b_1 + 0.1l_0 = 0.2 \times 1.35 + 0.1 \times 3.85 = 0.655\text{m}$$

$$b_{\text{eff},2} = 0.2b_2 + 0.1l_0 = 0.2 \times 1.625 + 0.1 \times 3.85 = 0.71\text{m}$$

$$b_{\text{eff}} = b_1 + b_2 + b_w = 0.655 + 0.71 + 0.3 = 1.665\text{m}$$

Step 3: Assume the neutral axis to be in the flange
Cross-Sectional analysis of reinforced concrete beam section for flexure
Prepared by: Concrete materials and structures chair

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| Step 4: | Assume the strain the tension reinforcement to be greater than the yield strain |
| Step 5: | Use the procedure of analysis of singly reinforced concrete sections to estimate neutral axis depth |

\[
\alpha_c = \frac{A_s f_y d}{f_{cd} b_d}
\]

\[
\alpha_c = \frac{\left(4 \times \pi \times 10^2\right) \times 347.83}{11.33 \times 1665 \times 457} = 0.0507
\]

Using the general design chart, \( k_x = 0.09 \), \( x = 37.017 \)

| Step 6: | Check if the assumption in step 3 is correct |

\( x = 37.017 < 120 \), the assumption is correct!

| Step 7: | Calculate the moment resistance |

Using the general design chart,

\( \varepsilon_{s1} = 25 \% \), the assumption is correct!

\( k_z = 0.987 \), \( z = 451.06 \)

\[ M = A_s f_y z = \left(4 \times \pi \times 10^2\right) \times 347.83 \times 451.06 = 197.16 kNm \]
**Cross-Sectional analysis of reinforced concrete beam section for flexure**

Prepared by: Concrete materials and structures chair

**Example 2 of 2**

Example 2.10: Compute the design resistance of the T–beam shown in the figure below.

The concrete and steel grades are C20/25 and S460 respectively. (Cover to stirrup = 25 mm and stirrup of diameter 8 c/c 200 was provided)

![T-beam diagram](image)

**Step 1:** Design Values (Changing the characteristic value to design value)

\[
d = 700 - (25 - 8 - 12) = 655 \text{ mm}
\]

\[
f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_C}
\]

\[
f_{yd} = \frac{f_{yk}}{\gamma_S}
\]

for persistent and transient design situation:

- $\gamma_C = 1.5$
- $\gamma_S = 1.15$

$\alpha_{cc} = 1$

**NB:** Take $\alpha_{cc} = 0.85$

\[
f_{cd} = \frac{0.85 \times 20}{1.5} = 11.33 \text{ MPa}
\]

\[
f_{yd} = \frac{460}{1.15} = 400 \text{ MPa}
\]

**Step 2:** Assume the neutral axis to be in the flange

**Step 3:** Assume the strain the tension reinforcement to be greater than the yield strain

**Step 4:** Use the procedure of analysis of singly reinforced concrete sections to estimate neutral axis depth

\[
\alpha_c = \frac{A_s f_{yd}}{f_{cd} b_d}
\]
Cross-Sectional analysis of reinforced concrete beam section for flexure
Prepared by: Concrete materials and structures chair

$$\alpha_c = \frac{(4 \times \pi \times 12^2) \times 400}{11.33 \times 500 \times 655} = 0.195$$

Using the general design chart, $k_x = 0.22$, $x = 144.1$

**Step 5:** Check if the assumption in step 3 is correct

$$x = 144.1 > 125$$, the assumption is not correct!

**Step 6:** Take the neutral axis to be below the flange and divide the section into two parts: Beam W and Beam F to simplify the analysis process.

**Step 7:** Take the rectangular stress strain relationship for the concrete under compression and calculate the moment resistance using force equilibrium.

**Beam F**

$$A_{sf} f_{yd} = f_{cd} (b_e - b_w) h_f$$

$$A_{sf} = \frac{f_{cd} (b_e - b_w) h_f}{f_{yd}} = \frac{11.33 \times (500 - 250) \times 125}{400} = 885.16 mm^2$$

$$M_{Rd,f} = A_{sf} f_{yd} \left( d - \frac{h_f}{2} \right)$$ or $$M_{Rd,f} = f_{cd} (b - b_w) h_f \left( d - \frac{h_f}{2} \right)$$

$$M_{Rd,f} = 11.33 \times (500 - 250) \times 125 \times (655 - 125/2) = 209.78 kNm$$

The force in the remaining steel area $A_{sw}$ is balanced by compression in the rectangular portion of the beam. (i.e. $A_{sw} = A_s - A_{sf}$)

**Beam W**

$$A_{sw} = A_s - A_{sf} = \left( 4 \times \pi \times 12^2 \right) - 885.16 = 924.4 mm^2$$
## Cross-Sectional analysis of reinforced concrete beam section for flexure

**Prepared by:** Concrete materials and structures chair

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$A_{sw}f_{yd} = 0.8f_{cd}b_w$</td>
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<tr>
<td>$x = \frac{A_{sw}f_{yd}}{0.8f_{cd}b_w}$</td>
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<tr>
<td>$x = \frac{924.4 \times 400}{0.8 \times 11.33 \times 250} = 163.18 \text{ mm}$</td>
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<tr>
<td>$M_{Rd,w} = A_{sw}f_{yd}(d - 0.4x)$ or $M_{Rd,w} = f_{cd} b_w(0.8x)(d - 0.4x)$</td>
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<tr>
<td>$M_{Rd,w} = 11.33 \times 250 \times 0.8 \times 163.18 \times (655 - 0.4 \times 655) = 218.06 \text{kNm}$</td>
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<tr>
<td>The total moment capacity of the section now becomes,</td>
<td></td>
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<tr>
<td>$M_{Rd} = M_{Rd,f} + M_{Rd,w}$</td>
<td></td>
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<tr>
<td>$M_{Rd} = 209.78 + 218.06 = 427.84 \text{kNm}$</td>
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**Step 8:** Calculate the strain in the tension reinforcement and check if the assumption is step 2 is correct. If it’s not found to be true, revise the procedure assuming the steel has not yielded.

\[
\frac{x}{d} = \frac{3.5}{3.5 + \varepsilon_s} \rightarrow \varepsilon_s = \frac{3.5d}{x} - 3.5 = \frac{3.5 \times 655}{163.18} - 3.5 = 10.54 > \varepsilon_{yd} \text{ ............ok!}
\]