CHAPTER 2. LIMIT STATE DESIGN FOR FLEXURE

2.1. INTRODUCTION

2.1.1. ANALYSIS VS. DESIGN

Two different types of problems arise in the study of reinforced concrete:

1. Analysis – Given a cross section, concrete strength, reinforcement size and location, and yield strength, compute the resistance or strength. In analysis there should be one unique answer.

2. Design – Given a factored design moment, select a suitable cross section, including dimensions, concrete strength, reinforcement, and so on. In design there are many possible solutions.

2.1.2. STATICS OF BEAM ACTION

A beam is a structural member that supports applied loads and its own weight primarily by internal moments and shears. Figure 2-1a shows a simple beam that supports its own dead weight, $\omega$ per unit length, plus a concentrated load, $P$. If the axial applied load, $N$, is equal to zero, as shown, the member is referred to as a beam. If $N$ is a compressive force, the member is called a beam-column. This chapter will be restricted to the very common case where $N = 0$.
The loads $\omega$ and P cause bending moments, distributed as shown in Figure 2-1b. The bending
moments is a load effect calculated from the loads by using the laws of statics. For a simply
supported beam of a given span and for a given set of loads $\omega$ and P, the moments are
independent of the composition and size of the beam.

At any section within the beam, the internal resisting moment, M, shown in Figure 2-1c is
necessary to equilibrate the bending moment. An internal resisting shear, V, also is required, as
shown.

The internal resisting moment, M, results from an internal compressive force, C, and an internal
tensile force, T, separated by a lever arm, jd, as shown in Figure 2-1d. Because there are no
external axial loads, summation of the horizontal forces gives

$$C - T = 0 \text{ or } C = T \quad (2-1)$$

If moments are summed about an axis through the point of application of the compressive force,
C, the moment equilibrium of the free body gives

$$M = T \times jd \quad (2-2)$$

Similarly, if moments are summed about the point of application of the tensile force, T,

$$M = C \times jd \quad (2-3)$$

Because $C = T$, these two equations are identical. Equations (2-1), (2-2) and (2-3) come directly
from statics and are equally applicable to beams made of steel, wood, or reinforced concrete.

The conventional elastic beam theory results in the equation $\sigma = My/I$, which, for an
uncracked, homogeneous rectangular beam without reinforcement, gives the distribution of
stresses shown in Figure 2-2.
Figure 2-2 – Elastic beam stresses and stress blocks

The stress diagram shown in Figure 2-2c and Figure 2-2d may be visualized as having a “volume”; hence, one frequently refers to the compressive stress block. The resultant compressive force C, which is equal to the volume of the compressive stress block in Figure 2-2d, is given by

\[ C = \frac{\sigma_{c(max)}}{2} \left( \frac{b}{2} \right) \left( \frac{h}{2} \right) \]  

(2-4)

In a similar manner, one could compute the force T from the tensile stress block. The forces C and T act through the centroids of the volumes of the respective stress blocks. In the elastic case, these forces act at \( h/3 \) above or below the neutral axis, so that \( jd = 2h/3 \).

From Equations (2-3) and (2-4) and Figure 2-2, we can write

\[ M = \sigma_{c(max)} \frac{bh}{4} \left( \frac{2h}{3} \right) \]  

(2-5)

\[ M = \sigma_{c(max)} \frac{bh^3}{12} \frac{h/2}{h/2} \]  

(2-6)

or, because

\[ I = \frac{bh^3}{12} \]  

(2-7)

And
\[ y_{\text{max}} = h/2 \]  

(2-8)

It follows that

\[ M = \frac{\sigma_{c(\text{max})} l}{y_{\text{max}}} \]  

(2-9)

Thus, for the elastic case, identical answers are obtained from the traditional beam stress equation (2-9), and when the stress block concept is used in equation (2-5).

The elastic beam theory in equation (2-9) is not used in the design of reinforced concrete beams, because the compressive stress-strain relationship for concrete becomes nonlinear at higher strain values. What is even more important is that concrete cracks at low tensile stresses, making it necessary to provide steel reinforcement to carry the tensile force, T.

### 2.2. DISTRIBUTION OF STRAINS AND STRESSES ACROSS A SECTION IN BENDING

The theory of bending for reinforced concrete assumes that the concrete will crack in the regions of tensile strains and that, after cracking, all the tension is carried by the reinforcement. It is also assumed that plane sections of a structural member remain plane after straining, so that across the section there must be a linear distribution of strains.

![Figure 2-3 – Singly reinforced rectangular beam](image)

Figure 2-3 shows the cross-section of a member subjected to bending, and the resultant strain diagram, together with three different types of stress distribution in the concrete:

1. The triangular stress distribution applies when the stresses are very nearly proportional to the strains, which generally occurs at the loading levels encountered under working conditions and is, therefore, used at the serviceability limit state.

2. The rectangular-parabolic stress block represents the distribution at failure when the compressive strains are within the plastic range, and it is associated with the design for the ultimate limit state.
3. The equivalent rectangular stress block is a simplified alternative to the rectangular parabolic distribution.

2.3. ULTIMATE LIMIT STATE FOR FLEXURE

2.3.1. BASIC ASSUMPTIONS FOR FLEXURE AT THE ULS

The theory of flexure for reinforced concrete is based on three basic assumptions, which are sufficient to allow one to calculate the moment resistance of a beam.

1. Sections perpendicular to the axis of bending that are plane before bending remain plane after bending.
2. The strain in the reinforcement is equal to the strain in the concrete at the same level.
3. The stresses in the concrete and reinforcement can be computed from the strains by using stress-strain curves for concrete and steel.
4. The tensile strength of the concrete is ignored.

The first of these is the traditional “plane sections remain plane” assumption made in the development of flexural theory for beams constructed with any material. The second assumption is necessary, because the concrete and the reinforcement must act together to carry load. This assumption implies a perfect bond between the concrete and the steel.

However, the assumptions are not strictly true. The deformations within a section are very complex, and, locally, plane sections do not remain plane. Nor, due to local bond slip, are the strains in the concrete exactly the same as those in the steel. Nevertheless, on average, the assumptions are correct, and are certainly sufficiently true for practical purposes for design of normal members.

2.3.2. POSSIBLE RANGE OF STRAIN DISTRIBUTIONS AT ULS

The possible range of strain distributions given in EN 1992-1-1-2004 is shown in Figure 2-4.
2.3.3. LIMITING COMPRESSIVE STRAINS AT ULS

It is universal to define failure of concrete in compression by means of a limiting compressive strain. The formulation of the limit varies from code to code, for example the American Concrete Institute code, ACI 318, uses a limit of 0.003, while the UK code BS 8110 uses 0.0035. For concrete strengths not exceeding 50 N/mm², the Eurocode adopts values of 0.0035 for flexure.
and for combined bending and axial load where the neutral axis remains within the section, and a limit of between 0.0035 and 0.002 for sections loaded so that the whole section is in compression.

The logic behind the reduction in the strain limit for axial compression is that, in axial compression, failure will occur at the strain corresponding to the attainment of the maximum compressive stress. This is 0.002 for concrete strengths not exceeding 50 N/mm$^2$. In flexure, considerably higher strains can be reached before the maximum capacity of the section is reached, and the value of 0.0035 has been obtained empirically.

2.4. TYPES OF FLEXURAL FAILURES

There are three types of flexural failures of reinforced concrete sections: tension, compression and balanced failures. These three types of failures may be discussed to choose the desirable type of failure from the three, in case failure is imminent.

A. Tension Failure

If the steel content $A_s$ of the section is small, the steel will reach $f_{yd}$ before the concrete reaches its maximum strain of $\varepsilon_{cu}$. With further increase in loading, the steel force remains constant at $f_{yd}$ $A_s$, but results a large plastic deformation in the steel, wide cracking in the concrete and large increase in compressive strain in the extreme fiber of concrete. With this increase in strain the stress distribution in the concrete becomes distinctly non-linear resulting in increase of the mean stress. Because equilibrium of internal forces should be maintained, the depth of the N.A decreases, which results in the increment of the lever arm $z$. The flexural strength is reached when concrete strain reaches $\varepsilon_{cu}$. This phenomenon is shown in Figure 2-6. This type of failure is preferable and is used for design.

![Figure 2-6 – Tension Failure](image)

B. Compression Failure
If the steel content $A_s$ is large, the concrete may reach its capacity before steel yields. In such a case the N.A depth increases considerably causing an increase in compressive force. Again the flexural strength of the section is reached $\varepsilon_{cu}$. The section fails suddenly in a brittle fashion. This phenomenon is shown in Figure 2-7.

![Figure 2-7 – Compression Failure](image)

**C. Balanced Failure**

At balanced failure the steel reaches $f_{yd}$ and the concrete reaches a strain of $\varepsilon_{cu}$ simultaneously. This phenomenon is shown in Figure 2-8.

![Figure 2-8 – Balanced Failure](image)

### 2.5. ANALYSIS OF BEAMS FOR FLEXURE AT THE ULS

Two requirements are satisfied throughout the flexural analysis and design of reinforced concrete beams and columns:
1. Stress and strain compatibility: The stress at any point in a member must correspond to the strain at that point
2. Equilibrium: Internal forces must balance the external load effects

2.5.1. ANALYSIS OF SINGLY REINFORCED BEAM SECTIONS

2.5.1.1. General procedure

The general procedure of analysis of singly reinforced concrete beams for its flexural resistance according to EN 1992-1-1-2004 is as follows.

- **Step 1:** Assume the type of failure

  From section 2.4, there are three possible types of failure for reinforced concrete beams under flexure. These are compression failure, tension failure and balanced failure.

- **Step 2:** Draw the strain profile corresponding to the type of failure

  a. Tension failure    b. Compression failure    c. Balanced failure

- **Step 3:** Take any of the three possible stress strain relationships for concrete described in chapter 1 to define the stress block

  a. Using parabola-rectangle diagram
b. Using Bi-linear diagram

c. Using rectangular diagram

- **Step 4:** Apply condition of equilibrium to the given stress block and conditions of compatibility to the strain profile to estimate the neutral axis depth

- **Step 5:** Calculate the unknown strain and check if the assumed type of failure is correct

- **Step 6:** If the assumption is correct, apply the moment equilibrium to the stress block and estimate the moment capacity

- **Step 7:** If it is not correct, assume another type of failure and repeat steps 2 to step 6 until the assumption is proven to be true

### 2.5.1.2. Simplified equations for moment and force equilibrium from stress – strain relationship

- **Assumptions**
  1. The section is rectangular with width ‘b’ and effective depth ‘d’
  2. Cylindrical compressive strength of the concrete is less than 50 MPa
  3. Stresses are in $\% \times 10^{-3}$
  4. $k_x = \frac{x}{d}$
5. $\varepsilon_{cm}$ is the compression strain at the ultimate fiber in the compressed region of the section

6. $d$ is effective depth of the cross section defined as the distance from the center of the tensile reinforcement bars to the top most compressed fiber

➢ **Force equilibrium**

$$C_c = \alpha_c f_{cd} bd \quad \text{and} \quad T_s = A_s f_{yd}$$

➢ **Moment equilibrium**

$$M = \alpha_c f_{cd} bd^2 (1 - \beta_c) \quad \text{or} \quad M = A_s f_{yd} d (1 - \beta_c)$$

➢ **Values of $\alpha_c$ and $\beta_c$**

1. **Using Parabolic rectangular stress – strain relationship**
   a) For $0 \leq \varepsilon_c \leq \varepsilon_{c2}$

   $$\alpha_c = \varepsilon_{cm} \left[ \frac{6 - \varepsilon_{cm}}{12} \right] k_x$$

   $$\beta_c = k_x \left[ \frac{8 - \varepsilon_{cm}}{4(6 - \varepsilon_{cm})} \right]$$

   b) For $\varepsilon_{c2} \leq \varepsilon_c \leq \varepsilon_{cu2}$

   $$\alpha_c = k_x \left( \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right)$$

   $$\beta_c = k_x \left[ \frac{\varepsilon_{cm} (3\varepsilon_{cm} - 4) + 2}{2\varepsilon_{cm} (3\varepsilon_{cm} - 2)} \right]$$

2. **Using Bi-linear stress-stress relationship**
   a) For $0 \leq \varepsilon_c \leq \varepsilon_{c3}$

   $$\alpha_c = \frac{2\varepsilon_{cm} k_x}{7}$$

   $$\beta_c = \frac{k_x}{3}$$

   b) For $\varepsilon_{c3} \leq \varepsilon_c \leq \varepsilon_{cu3}$

   $$\alpha_c = k_x \left( \frac{2\varepsilon_{cm} - 1.75}{2\varepsilon_{cm}} \right)$$
\[ \beta_c = k_x \left( \frac{\varepsilon_{cm} (3\varepsilon_{cm} - 5.25) + 3.0625}{3\varepsilon_{cm} (2\varepsilon_{cm} - 1.75)} \right) \]

3. Using simplified rectangular block

\[ \alpha_c = 0.8k_x \]
\[ \beta_c = 0.4k_x \]

2.5.1.3. Simplified procedure for analysis of singly reinforced beam sections using the parabolic – rectangular stress – strain relationship

- **Step 1:** Assume the type of failure
  - Tension Failure
    - Rupture of steel \( \varepsilon_s = 25 \)
    - The strain in the steel exceeded the yield strain and the most compressed concrete has reached the crushing strain. \( \varepsilon_{cm} = 3.5 \) and \( \varepsilon_s > \varepsilon_{yd} \)
  - Compression Failure
    - Assume \( \varepsilon_{cm} = 3.5 \) and \( \varepsilon_s < \varepsilon_{yd} \)

- **Step 2:** Draw the strain profile corresponding to the type of failure and use the similarity of triangles to develop a relationship between the unknown strain and the neutral axis.
- **Step 3:** Use the equation of alpha corresponding to the assumption in step 1 and the relationship developed in step 2 to calculate the unknown strain and neutral axis depth.
- **Step 4:** Check if the assumption in step 1 is correct and if it is, proceed to step 8. If the assumption is not correct repeat step 1 to 6 with another assumption.
- **Step 5:** Calculate the value of beta
- **Step 6:** Calculate the moment resistance

2.5.2. ANALYSIS OF DOUBLY REINFORCED BEAM SECTIONS

Occasionally, beam sections are designed to have both tension reinforcement and compression reinforcement. These are referred to as doubly reinforced sections. Two cases where compression reinforcement is used frequently are the negative bending region of continuous beams and mid-span regions of long-span or heavily loaded beams where deflections need to be controlled.

2.5.2.1. Reasons for providing compression reinforcement

There are four primary reasons for using compression reinforcement in beams:

1. **Reduced sustained-load deflections.** First and most important, the addition of compression reinforcement reduces the long-term deflections of a beam subjected to sustained loads. Creep of the concrete in the compression zone transfers load from the concrete to the
compression steel, reducing the stress in the concrete. Because of the lower compression stress in the concrete, it creeps less, leading to a reduction in sustained-load deflections.

2. **Increased ductility.** The addition of compression reinforcement causes a reduction in the depth of the compression stress block and the strain in the tension reinforcement at failure increases resulting in more ductile behavior.

3. **Change of mode of failure from compression to tension.** When enough compression steel is added to a beam, the compression zone is strengthened sufficiently to allow the tension steel to yield before the concrete crushes. The beam then displays a ductile mode of failure.

4. **Fabrication ease.** When assembling the reinforcing cage for a beam, it is customary to provide small bars in the corner of the stirrups to hold the stirrups in place in the form and also to help anchor the stirrups. If developed properly, these bars in effect are compression reinforcement, although they generally are disregarded in design, because they have only a small effect on the moment strength.

### 2.5.2.2. General procedure

The general procedure of analysis of doubly reinforced concrete beams for its flexural resistance according to EN 1992-1-1-2004 is as follows.

- **Step 1:** Assume the type of failure
- **Step 2:** Draw the strain profile corresponding to the type of failure
- **Step 3:** Assume the strain in the negative reinforcement either to be greater than the yield strain or to be less than the yield strain
- **Step 4:** Take any of the three possible stress strain relationships for concrete described in chapter 1 to define the stress block
- **Step 5:** Take the stress strain relationship for the reinforcement bar
- **Step 6:** Apply condition of equilibrium to the given stress block and conditions of compatibility to the strain profile to estimate the neutral axis depth
- **Step 7:** Calculate the strain in the compression reinforcement bars and check if the assumption is step 3 in correct.
- **Step 8:** If the assumption is true, proceed to step 9, otherwise revise the assumption in step 3 and repeat steps 4 to 7.
- **Step 9:** Calculate the unknown strain and check if the assumed type of failure is correct. If the assumption is correct, proceed to step 10, otherwise repeat step 1 to 9 assuming another type of failure mode.
- **Step 10:** Apply the moment equilibrium to the stress block and estimate the moment capacity
2.5.3. ANALYSIS OF FLANGED SECTIONS

2.5.3.1. Introduction
In the floor system shown in Figure 2-9, the slab is assumed to carry the loads in one direction to beams that carry them in the perpendicular direction. During construction, the concrete in the columns is placed and allowed to harden before the concrete in the floor is placed. In the next construction operation, concrete is placed in the beams and slab in a monolithic pour. As a result, the slab serves as the top flange of the beams, as indicated by the shading in Figure 2-9. Such a beam is referred to as a T-beam. The interior beam, AB, has flange on both sides. The spandrel beam, CD, with a flange on one side only, is often referred to as an inverted L-beam.

![Figure 2-9 – T-beams in a one-way beam and slab floor](image)

[Image: Figure 2-9 – T-beams in a one-way beam and slab floor]
An exaggerated deflected view of the interior beam is shown in Figure 2-10. This beam develops positive moments at midspan (section A-A) and negative moments over the supports (section B-B). At midspan, the compression zone is in the flange, as shown in Figure 2-10b and Figure 2-10d. Generally, it is rectangular, as shown Figure 2-10b, although, in very rare cases for typical reinforced concrete construction, the neutral axis may shift down into the web, giving a T-shaped compression zone, as shown in Figure 2-10d. At the support, the compression zone is at the bottom of the beam and is rectangular, as shown in Figure 2-10c.

2.5.3.2. **Effective flange width**

The forces acting on the flange of a simply supported T-beam are illustrated in Figure 2-11. At the support, there are no longitudinal compressive stresses in the flange, but at midspan, the full width is stressed in compression. The transition requires horizontal shear stresses on the web-flange interface as shown in Figure 2-11. As a result there is a “shear-lag” effect, and the portions of the flange closest to the web are more highly stressed than those portions farther away, as shown in Figure 2-11 and Figure 2-12.
Figure 2-11 – Actual flow of forces on a T-beam flange

Figure 2-12a shows the distribution of the flexural compressive stresses in a slab that forms the flanges of a series of parallel beam at a section of maximum positive moment. The compressive stress is a maximum over each web, dropping between the webs. When analyzing and designing the section for positive moments, an effective compression flange width is used (Figure 2-12b). When this width, $b_e$, is stressed uniformly, it will give approximately the same compression force that actually is developed in the full width of the compression zone.
Figure 2-12 – Effective width of T-beams

According to EN 1992-1-1-2004, in T beams the effective flange width, over which uniform conditions of stress can be assumed, depends on the web and flange dimensions, the type of loading, the span, the support conditions and the transverse reinforcement.

The effective width of flange should be based on the distance $l_0$ between points of zero moment, which may be obtained from Figure 2-13.

Note: The length of the cantilever, $l_3$, should be less than half the adjacent span and the ratio of adjacent spans should lie between 2/3 and 1.5.

The effective flange width parameters are shown in Figure 2-12 below.
The effective flange width $b_{\text{eff}}$ for a T beam or L beam may be derived as:

$$b_{\text{eff}} = \sum b_{\text{eff},j} + b_w \leq b$$  \hspace{1cm} (2-10)

where

$$b_{\text{eff},j} = 0.2b_j + 0.1l_0 \leq 0.2l_0$$  \hspace{1cm} (2-11)

and

$$b_{\text{eff},j} \leq b_j$$  \hspace{1cm} (2-12)

### 2.5.3.3. Procedure of analysis of flanged beam for flexure

#### a) Flanged beam subjected to negative moment

For a flanged beam with a negative moment, the compression zone will be the bottom rectangular part of the web, thus following the procedures for analysis of rectangular sections will be appropriate.

#### b) Flanged beam subjected to positive moment

If a flanged beam is subjected to positive moment, the neutral axis might remain within the flange of the beam or it might be in the web of the beam.

For the case where the neutral axis remains in the flange, the section may be treated as a rectangular section, and the procedures of analysis of rectangular sections can be adopted. However, if the neutral axis is in the web of the beam, a different approach for analysis is necessary and in doing so, adopting the rectangular stress relationship for the concrete in compression will simplify the analysis.

The general procedure for the analysis of flanged beam subjected to positive moment according to EN 1992-1-1-2004 is as follows.
• **Step 1:** Assume the neutral axis to be in the flange

• **Step 2:** Assume the strain in the tension reinforcement to be greater than the yield strain

• **Step 3:** Use the procedure of analysis of singly reinforced concrete sections to estimate neutral axis depth

• **Step 4:** Check if the assumption in step 1 is correct

• **Step 5:** If the assumption is correct, estimate the moment resistance of the section using the procedures of singly reinforced concrete sections. If not correct, proceed to step 6.

• **Step 6:** Take the neutral axis to be below the flange and divide the section into two parts: Beam W and Beam F to simplify the analysis process.

• **Step 7:** Take the rectangular stress strain relationship for the concrete under compression and calculate the moment resistance using force equilibrium.

**Beam F**

\[ A_s f_{yd} = f_{cd}(b_e - b_w)h_f \]

\[ A_s = \frac{f_{cd}(b_e - b_w)h_f}{f_{yd}} \]

\[ M_{Rd,f} = A_s f_{yd} \left( d - \frac{h_f}{2} \right) \text{ or } M_{Rd,f} = f_{cd}(b - b_w)h_f \left( d - \frac{h_f}{2} \right) \]

The force in the remaining steel area \( A_{sw} \) is balanced by compression in the rectangular portion of the beam. (i.e. \( A_{sw} = A_s - A_s \))

**Beam W**

\[ A_{sw} f_{yd} = 0.8xf_{cd}b_w \]

\[ x = \frac{A_{sw} f_{yd}}{0.8f_{cd}b_w} \]

\[ M_{Rd,w} = A_{sw} f_{yd}(d - 0.4x) \text{ or } M_{Rd,w} = f_{cd}b_w(0.8x)(d - 0.4x) \]
The total moment capacity of the section now becomes,

\[ M_{Rd} = M_{Rd,f} + M_{Rd,w} \]

- **Step 8:** Calculate the strain in the tension reinforcement and check if the assumption is step 2 is correct. If it’s not found to be true, revise the procedure assuming the steel has not yielded.

### 2.6. DESIGN OF BEAMS FOR FLEXURE AT ULS

#### 2.6.1. INDICATIVE STRENGTH CLASSES FOR DURABILITY

The choice of adequately durable concrete for corrosion protection of reinforcement and protection of concrete attack requires consideration of the composition of concrete. This may result in a higher compressive strength of the concrete than is required for structural design. The relationship between concrete strength classes and environmental classes (Table 2-1) may be described by indicative strength classes.

<table>
<thead>
<tr>
<th>Exposure Classes according to Table 4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrosion</td>
</tr>
<tr>
<td>Carbonation-induced corrosion</td>
</tr>
<tr>
<td>Chloride-induced corrosion</td>
</tr>
<tr>
<td>Chloride-induced corrosion from sea-water</td>
</tr>
<tr>
<td>XC1</td>
</tr>
<tr>
<td>C20/25</td>
</tr>
<tr>
<td>Indicative Strength Class</td>
</tr>
<tr>
<td>C20/25</td>
</tr>
</tbody>
</table>

#### 2.6.2. CONCRETE COVER

It is necessary to have cover (concrete between the surface of the slab or beam and the reinforcing bars) for four primary reasons:

1. To bond the reinforcement to the concrete so that the two elements act together. The efficiency of the bond increases as the cover increases.
2. To protect the reinforcement against corrosion.
3. To protect the reinforcement from strength loss due to overheating in the case of fire.
4. Additional cover sometimes is provided on the top of slabs, particularly in garages and factories, so that abrasion and wear due to traffic will not reduce the cover below that required for structural and other purposes.

The concrete cover is the distance between the surface of the reinforcement closest to the nearest concrete surface (including links and stirrups and surface reinforcement where relevant) and the nearest concrete surface.
Concrete cover according to EN 1992-1-1 and EN 1992-1-2

The nominal cover is defined as a minimum cover, $c_{\text{min}}$, plus an allowance in design for deviation, $\Delta c_{\text{dev}}$:

$$c_{\text{nom}} = c_{\text{min}} + \Delta c_{\text{dev}} \quad (2-13)$$

where $c_{\text{min}}$ should be set to satisfy the requirements below:

- Safe transmission of bond forces
- Durability
- Fire resistance

and $\Delta c_{\text{dev}}$ is an allowance which should be made in the design for deviations from the minimum cover. It should be taken as 10 mm, unless fabrication (i.e. construction) is subjected to a quality assurance system, in which case it is permitted to reduce $\Delta c_{\text{dev}}$ to 5 mm.

$$c_{\text{min}} = \max\{c_{\text{min,b}}, c_{\text{min,dur}}, 10 \text{ mm}\}$$

1. **Minimum cover for bond, $c_{\text{min,b}}$**

The minimum cover to ensure adequate bond should not be less than the bar diameter, unless the aggregate size is over 32 mm. If the aggregate size is over 32 mm, $c_{\text{min,b}}$ should be increased by 5 mm.

2. **Minimum cover for durability**

EC-2 leaves the choice of $C_{\text{min,dur}}$ to countries, but gives the following recommendation:

The value of $C_{\text{min,dur}}$ depends on the “structural class”, which has to be determined first. If the specified service life is 50 years, the structural class is defined as 4. The “structural class” can be modified in case of the following conditions:

- The service life is 100 years instead of 50 years
- The concrete strength is higher than necessary
- Slabs (position of reinforcement not affected by construction process)
- Special quality control measures apply

The finally applying service class can be calculated with Table 4.3 N but the recommended minimum structural class is 1.
<table>
<thead>
<tr>
<th>Class designation</th>
<th>Description of the environment</th>
<th>Informative examples where exposure classes may occur</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 No risk of corrosion or attack</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X0</td>
<td>For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack For concrete with reinforcement or embedded metal: very dry</td>
<td>Concrete inside buildings with very low air humidity</td>
</tr>
<tr>
<td>2 Corrosion induced by carbonation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XC1</td>
<td>Dry or permanently wet</td>
<td>Concrete inside buildings with low air humidity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Concrete permanently submerged in water</td>
</tr>
<tr>
<td>XC2</td>
<td>Wet, rarely dry</td>
<td>Concrete surfaces subject to long-term water contact</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Many foundations</td>
</tr>
<tr>
<td>XC3</td>
<td>Moderate humidity</td>
<td>Concrete inside buildings with moderate or high air humidity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>External concrete sheltered from rain</td>
</tr>
<tr>
<td>XC4</td>
<td>Cyclic wet and dry</td>
<td>Concrete surfaces subject to water contact, not within exposure class XC2</td>
</tr>
<tr>
<td>3 Corrosion induced by chlorides</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XD1</td>
<td>Moderate humidity</td>
<td>Concrete surfaces exposed to airborne chlorides</td>
</tr>
<tr>
<td>XD2</td>
<td>Wet, rarely dry</td>
<td>Swimming pools</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Concrete components exposed to industrial waters containing chlorides</td>
</tr>
<tr>
<td>XD3</td>
<td>Cyclic wet and dry</td>
<td>Parts of bridges exposed to spray containing chlorides</td>
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<td></td>
<td></td>
<td>Pavements</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Car park slabs</td>
</tr>
<tr>
<td>4 Corrosion induced by chlorides from sea water</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XS1</td>
<td>Exposed to airborne salt but not in direct contact with sea water</td>
<td>Structures near to or on the coast</td>
</tr>
<tr>
<td>XS2</td>
<td>Permanently submerged</td>
<td>Parts of marine structures</td>
</tr>
<tr>
<td>XS3</td>
<td>Tidal, splash and spray zones</td>
<td>Parts of marine structures</td>
</tr>
<tr>
<td>5. Freeze/Thaw Attack</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XF1</td>
<td>Moderate water saturation, without de-icing agent</td>
<td>Vertical concrete surfaces exposed to rain and freezing</td>
</tr>
<tr>
<td>XF2</td>
<td>Moderate water saturation, with de-icing agent</td>
<td>Vertical concrete surfaces of road structures exposed to freezing and airborne de-icing agents</td>
</tr>
<tr>
<td>XF3</td>
<td>High water saturation, without de-icing agents</td>
<td>Horizontal concrete surfaces exposed to rain and freezing</td>
</tr>
<tr>
<td>XF4</td>
<td>High water saturation with de-icing agents or sea water</td>
<td>Road and bridge decks exposed to de-icing agents</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Concrete surfaces exposed to direct spray containing de-icing agents and freezing</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Splash zone of marine structures exposed to freezing</td>
</tr>
<tr>
<td>6. Chemical attack</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XA1</td>
<td>Slightly aggressive chemical environment according to EN 206-1, Table 2</td>
<td>Natural soils and ground water</td>
</tr>
<tr>
<td>XA2</td>
<td>Moderately aggressive chemical environment according to EN 206-1, Table 2</td>
<td>Natural soils and ground water</td>
</tr>
<tr>
<td>XA3</td>
<td>Highly aggressive chemical environment according to EN 206-1, Table 2</td>
<td>Natural soils and ground water</td>
</tr>
</tbody>
</table>
3. Minimum cover for fire resistance

Rather than giving the minimum cover, the tabular method is based on nominal axis distance, see Fig. this is the distance from the center of the main reinforcing bar to the surface of the member. The designer should ensure that

$$a \geq c_{\text{nom}} + \phi_{\text{link}} + \phi_{\text{bar}} / 2$$  \hspace{1cm} (2-14)

\[\text{Table 4.3N: Recommended structural classification}\]

<table>
<thead>
<tr>
<th>Structural Class</th>
<th>Exposure Class according to Table 4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X0</td>
</tr>
<tr>
<td>Service Life of 100 years</td>
<td>increase class by 2</td>
</tr>
<tr>
<td>Strength Class</td>
<td>≥ C30/37</td>
</tr>
<tr>
<td>Member with slab geometry (position of reinforcement not affected by construction process)</td>
<td>reduce class by 1</td>
</tr>
<tr>
<td>Special Quality Control ensured</td>
<td>reduce class by 1</td>
</tr>
</tbody>
</table>

\[\text{Table 4.4N: Values of minimum cover, } c_{\min,\text{dun}}, \text{ requirements with regard to durability for reinforcement steel}\]

<table>
<thead>
<tr>
<th>Environmental Requirement for }c_{\min} (mm)</th>
<th>Exposure Class according to Table 4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structural Class</td>
<td>X0</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
</tr>
</tbody>
</table>
Table 2-2 – Minimum dimensions and axis distances for simply supported beams made with reinforced and prestressed concrete

<table>
<thead>
<tr>
<th>Standard fire resistance</th>
<th>Minimum dimensions (mm)</th>
<th>Web thickness $d_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Possible combinations of $a$ and $b_{min}$ where $a$ is the average axis distance and $b_{min}$ is the width of beam</td>
<td>Class WA</td>
</tr>
<tr>
<td>R 30</td>
<td>$b_{min}= 80$ $a = 25$ 120 20 160 300 400 110</td>
<td>80 80 80</td>
</tr>
<tr>
<td>R 60</td>
<td>$b_{min}= 120$ $a = 40$ 200 30 25</td>
<td>100 80 100</td>
</tr>
<tr>
<td>R 90</td>
<td>$b_{min}= 155$ $a = 55$ 40 40 35 35</td>
<td>110 100 100</td>
</tr>
<tr>
<td>R 120</td>
<td>$b_{min}= 200$ $a = 65$ 300 50 60 70</td>
<td>130 120 120</td>
</tr>
<tr>
<td>R 180</td>
<td>$b_{min}= 240$ $a = 80$ 400 60 60</td>
<td>150 150 140</td>
</tr>
<tr>
<td>R 240</td>
<td>$b_{min}= 280$ $a = 90$ 500 70 70</td>
<td>170 170 160</td>
</tr>
<tr>
<td>$a_{sq} = a + 10mm$ (see note below)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For prestressed beams the increase of axis distance according to 5.2(5) should be noted.

$a_{sq}$ is the axis distance to the side of beam for the corner bars (or tendon or wire) of beams with only one layer of reinforcement. For values of $b_{min}$ greater than that given in Column 4 no increase of $a_{sq}$ is required.

* Normally the cover required by EN 1992-1-1 will control.
Table 2-3 – Minimum dimensions and axis distances for continuous beams made with reinforced and prestressed concrete

<table>
<thead>
<tr>
<th>Standard fire resistance</th>
<th>Minimum dimensions (mm)</th>
<th>Web thickness bw</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Possible combinations of a and bmin where a is the average axis distance and bmin is the width of beam</td>
<td>Class WA</td>
</tr>
<tr>
<td>R 30</td>
<td>bmin= 80</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>a = 15*</td>
<td>12*</td>
</tr>
<tr>
<td>R 60</td>
<td>bmin= 120</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>a = 25</td>
<td>12*</td>
</tr>
<tr>
<td>R 90</td>
<td>bmin= 150</td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>a = 35</td>
<td>25</td>
</tr>
<tr>
<td>R 120</td>
<td>bmin= 200</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>a = 45</td>
<td>35</td>
</tr>
<tr>
<td>R 180</td>
<td>bmin= 240</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>a = 60</td>
<td>50</td>
</tr>
<tr>
<td>R 240</td>
<td>bmin= 280</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>a = 75</td>
<td>60</td>
</tr>
</tbody>
</table>

a_sc = a + 10mm (see note below)

For prestressed beams the increase of axis distance according to 5.2(5) should be noted.

a_sc is the axis distance to the side of beam for the corner bars (or tendon or wire) of beams with only one layer of reinforcement. For values of bmin greater than that given in Column 3 no increase of a_sc is required.

* Normally the cover required by EN 1992-1-1 will control.

2.6.3. MINIMUM AND MAXIMUM AREA OF REINFORCEMENT

Minimum and Maximum area of reinforcement according to EN 1992-1-1-2004

The area of longitudinal tension reinforcement should not be taken as less than A_{b,min}

\[ A_{b,\text{min}} = 0.26 \frac{f_{\text{am}}}{f_{yk}} b_d \]

but not less than 0.0013b_d \hspace{1cm} (2-15)

Where:
denotes the mean width of the tension zone; for a T-beam with the flange in compression, only the width of the web is taken into account in calculating the value of $b_t$.

$f_{ctm}$ should be determined with respect to the relevant strength class according to Table 3.1. of Eurocode

The cross-sectional area of tension or compression reinforcement should not exceed $A_{s,\text{max}}$ outside lap locations.

The value of $A_{s,\text{max}}$ for beams for use in a country may be found in its National Annex. The recommended value is $0.04A_b$.

2.6.4. SPACING OF BARS

Spacing of bars according to EN 1992-1-1-2004

The clear distance (horizontal and vertical) between individual parallel bars or horizontal layers of parallel bars should be not less than

$$\max\left\{\frac{k_1}{d} \cdot \text{bar diameter}, \frac{d_g + k_2}{20} \right\}$$

Where $d_g$ is the maximum size of aggregate

The recommended values of $k_1$ and $k_2$ are 1 and 5 mm respectively

2.6.5. DESIGN OF SINGLY REINFORCED BEAM SECTIONS

2.6.5.1. Limit to the use of singly reinforced sections

At the ultimate limit state it is important that member sections in flexure should be ductile and that failure should occur with the gradual yielding of the tension steel and not by a sudden catastrophic compression failure of the concrete. Also, yielding of the reinforcement enables the formation of plastic hinges so that redistribution of maximum moments can occur, resulting in a safer and more economical structure.

To ensure rotation of the plastic hinges with sufficient yielding of the tension steel and also to allow for other factors such as the strain hardening of the steel, Clause 5.5 in EN 1992-1-1 give limits to the neutral axis depth at the ultimate limit state as a function of the amount of redistribution carried out in the analysis.

$$\delta \leq k_1 + k_2 \frac{x_u}{d} \quad \text{for } f_{ck} \leq 50 \text{MPa}$$

$$\delta \leq k_3 + k_4 \frac{x_u}{d} \quad \text{for } f_{ck} > 50 \text{MPa}$$

(2-16)
Where:

\( \delta \) is the ratio of the redistributed moment to the elastic bending moment

\( x_u \) is the depth of the neutral axis at the ultimate limit state after redistribution

The values of \( k_1 \), \( k_2 \), \( k_3 \), and \( k_4 \) for use in a country may be found in its National Annex. The recommended value is

\[
\begin{align*}
    k_1 &= 0.44 \\
    k_2 &= 1.25 \left(0.6 + 0.0014 \varepsilon_{cu2}\right) \\
    k_3 &= 0.54 \\
    k_4 &= 1.25 \left(0.6 + 0.0014 \varepsilon_{cu2}\right)
\end{align*}
\]

Substituting the recommended values for \( f_{ck} \leq 50 \text{ MPa} \) gives \( x_u/d = 0.448 \)

### 2.6.5.2. Design procedure

The general procedure for the design of singly reinforced beams according to EN 1992-1-1 using Design Chart is as follows.

- **Step 1:** Take a strain distribution that results a ductile failure

  ![Strain Distribution Diagram]

- **Step 2:** Use equilibrium of forces to estimate the value of \( k_x \)

  For \( \varepsilon_{cm} > \varepsilon_{c2} \), \( \alpha_c = k_x \left( \frac{3 \varepsilon_{cm} - 2}{3 \varepsilon_{cm}} \right) \)

  Substituting \( \varepsilon_{cm} = 3.5 \), \( \alpha_c = k_x \left( \frac{8.5}{10.5} \right) \)

  For \( \varepsilon_{cm} > \varepsilon_{c2} \), \( \beta_c = k_x \left[ \frac{\varepsilon_{cm} (3 \varepsilon_{cm} - 4) + 2}{2 \varepsilon_{cm} (3 \varepsilon_{cm} - 2)} \right] \)
Substituting \( e_{cm} = 3.5 \), \( \beta_c = k_x \left( \frac{24.75}{59.5} \right) \)

From equilibrium of moments,

\[ M = \alpha_c f_{cd} b d^2 \left( 1 - \beta_c \right) \]

Substituting the values of \( \alpha_c \) and \( \beta_c \)

\[ M = k_x \left( \frac{8.5}{10.5} \right) f_{cd} b d^2 \left( 1 - k_x \left( \frac{24.75}{59.5} \right) \right) \]

\[ \frac{M}{f_{cd} b d^2} \left( \frac{10.5}{8.5} \right) = \left( k_x - k_x^2 \left( \frac{24.75}{59.5} \right) \right) \]

\[ k_x^2 \left( \frac{24.75}{59.5} \right) - k_x + \frac{M}{f_{cd} b d^2} \left( \frac{10.5}{8.5} \right) = 0 \]

Solving the above quadratic equations results the value of \( k_x \)

- **Step 3:** If \( k_x < k_{x,\text{lim}} \), then compression reinforcement is not required and

\[ A_s = \frac{M}{f_{yd} d (1 - \beta_c)} \]

If \( k_x > k_{x,\text{lim}} \), then compression reinforcement is required and the section is designed as a double reinforced section.

- **Step 4:** Check the minimum and maximum area of reinforcement

### 2.6.6. DESIGN OF DOUBLY REINFORCED SECTIONS

The general procedure for the design of doubly reinforced beams according to EN 1992-1-1 is using design chart is as follows.

- **Step 1:** Take a strain distribution that results a ductile failure
• **Step 2:** Assume the section as having two parts.

• **Step 3:** Use equilibrium of forces to estimate the area of tension and compression reinforcement.

For \( \varepsilon_{cm} > \varepsilon_{c2} \), \( \alpha_c = k_x \left( \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right) \)

Substituting \( \varepsilon_{cm} = 3.5 \), \( \alpha_c = k_x \left( \frac{8.5}{10.5} \right) \)

For \( \varepsilon_{cm} > \varepsilon_{c2} \), \( \beta_c = k_x \left[ \frac{\varepsilon_{cm} (3\varepsilon_{cm} - 4) + 2}{2\varepsilon_{cm} (3\varepsilon_{cm} - 2)} \right] \)

Substituting \( \varepsilon_{cm} = 3.5 \), \( \beta_c = k_x \left( \frac{24.75}{59.5} \right) \)

Single reinforced part:

Take, \( k_x = 0.448 \),
\[
\alpha_c^* = 0.448 \left( \frac{8.5}{10.5} \right) = 0.363 \quad \beta_c^* = 0.448 \left( \frac{24.75}{59.5} \right) = 0.186
\]

\[
M^* = \alpha_c^* f_{cd} bd^2 \left( 1 - \beta_c^* \right)
\]

\[
M^* = 0.363 f_{cd} bd^2 \left( 1 - 0.186 \right)
\]

\[
M^* = 0.295 f_{cd} bd^2
\]

\[
M^* = A_s^* f_{yd} d \left( 1 - \beta_c^* \right) \rightarrow A_s^* = \frac{M^*}{f_{yd} d \left( 1 - \beta_c^* \right)} = \frac{M^*}{f_{yd} d \left( 1 - 0.186 \right)} = \frac{M^*}{0.814 f_{yd} d}
\]

Double reinforced part:

\[
M - M^* = f_{s2} A_{s2} (d - d_2) \rightarrow A_{s2} = \frac{M - M^*}{f_{s2} (d - d_2)}
\]

\[
M - M^* = (A_s - A_s^*) f_{yd} (d - d_2) \rightarrow A_s = \frac{M - M^*}{f_{yd} (d - d_2)} + A_s^*
\]

- **Step 4:** Check the minimum and maximum area of reinforcement

### 2.6.7. DESIGN OF FLANGED BEAMS

a) **Flanged beam subjected to negative moment**

For a flanged beam with a negative moment, the compression zone will be the bottom rectangular part of the web, thus following the procedures for design of rectangular sections will be appropriate.

b) **Flanged beam subjected to positive moment**

If a flanged beam is subjected to positive moment, the neutral axis might remain within the flange of the beam or it might be in the web of the beam.

For the case where the neutral axis remains in the flange, the section may be treated as a rectangular section with width \(b\) as the width of the flange, and the procedures of analysis of rectangular sections can be adopted. However, if the neutral axis is in the web of the beam, a different approach for design is necessary and in doing so, adopting the rectangular stress relationship for the concrete in compression will simplify the procedure.
The general procedure for the design of flanged beam subjected to positive moment according to EN 1992-1-1 is as follows.

- **Step 1:** Take a strain distribution that results a ductile failure

  ![Image of strain distribution](image)

- **Step 2:** Assume the neutral axis to be in the flange and use procedure of singly reinforced sections to calculate the neutral axis depth

- **Step 3:** If the neutral axis is found to be in the flange, calculate the moment resistance of the section following the procedure for rectangular section. If the neutral axis is in the web of the section proceed to step 4

- **Step 4:** Divide the section into two parts: Beam W and Beam F to simplify the design process.

  ![Image of beam sections](image)

  Beam F
  \[
  A_{sf} f_{yd} = f_{cd} (b_e - b_w) h_f
  \]

  \[
  A_{sf} = \frac{f_{cd} (b_e - b_w) h_f}{f_{yd}}
  \]

  \[
  M_{Rd,f} = A_{sf} f_{yd} \left( d - \frac{h_f}{2} \right) \text{ or } M_{Rd,f} = f_{cd} (b - b_w) h_f \left( d - \frac{h_f}{2} \right)
  \]
The total area of tension reinforcement becomes,

\[ A_s = A_{sw} + A_{sf} \]

- **Step 5:** Check the minimum and maximum area of reinforcement

### 2.7. ANALYSIS AND DESIGN OF ONE-WAY SLAB SYSTEM

#### 2.7.1. INTRODUCTION

Slabs are surface plane elements that bear loads transverse to their plain. Most of the times, slabs are statically indeterminate elements that consequently redistribute the stresses applied to them. This ability makes them highly secure against bending and shear failure.

![Types of slabs based on their load transfer mechanism](image)

**Figure 2-15 Types of slabs based on their load transfer mechanism**

1. **One-way slabs**: They are those either supported on the two out of four opposite sides or the longer span to short span ratio is at least equal to 2.
2. **Two-way slabs**: They are those supported on all four sides and the longer span to short span ratio is less than 2.

3. **Cantilever slabs**: They are those with a fixed support on only one out of four sides

![Figure 2-16 One-way slab supported by two beams](image)

In the design and analysis of one way slab systems a 1m strip of slab along the load transverse direction is considered as shown below.
2.7.2. ANALYSIS OF ONE-WAY SLABS

For one-way slab sections with under both a negative and positive bending moment follows the procedures analysis of rectangular sections discussed in section 2.5.1.3 and 2.5.2. The only exception is that the width of the slab considered is 1m as previously pointed out.

2.7.3. DESIGN OF ONE-WAY SLAB

2.7.3.1. Concrete cover

The cover requirement for bond and durability are the same for that of beam requirements discussed in section 2.6.1.

But for fire resistance the minimum dimension and cover requirements are given in EN 1992-1-2:2004 table 5.8.
2.7.3.2. Minimum and Maximum area of reinforcement (Flexural reinforcement)

For the minimum and the maximum steel percentages in the main direction sections 2.6.3 apply.

Secondary transverse reinforcement of not less than 20% of the principal reinforcement should be provided in one way slabs. In areas near supports transverse reinforcement to principal top bars is not necessary where there is no transverse bending moment.

The spacing of bars should not exceed $s_{\text{max,slabs}}$.

Note: The value of $s_{\text{max,slabs}}$ for use in a Country may be found in its National Annex. The recommended value is:

- for the principal reinforcement, $3h \leq 400$ mm, where $h$ is the total depth of the slab;
- for the secondary reinforcement, $3.5h \leq 450$ mm.

In areas with concentrated loads or areas of maximum moment those provisions become respectively:

- for the principal reinforcement, $2h \leq 250$ mm
- for the secondary reinforcement, $3h \leq 400$ mm.
2.7.3.3. Design of one-way slab
The same procedure carried out for design of singly reinforced rectangular beam sections (2.6.5.2) can be adopted for one-way slabs but the above discussed points must be adopted in the design procedures.